

Differentialrechnung

Differenzenquotient: Steigung der Sekante

$$\frac{f(x_1) - f(x_2)}{x_1 - x_2} = \frac{y_1 - y_2}{x_1 - x_2} = \frac{\Delta y}{\Delta x}$$

Differentialquotient: Steigung der Tangente

$$\lim_{x \rightarrow z} \frac{f(z) - f(x)}{z - x} = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h} = f'(x)$$

Ableitungsregeln:

- $f(x) = k$ $f'(x) = 0$
- $f(x) = x^q$ $f'(x) = qx^{q-1}$
- $f(x) = kx^q$ $f'(x) = kqx^{q-1}$
- $(f \pm g)' = f' \pm g'$

Produktregel:

$$(fg)' = f'g + fg'$$

Quotientenregel:

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

Kettenregel:

$$h(x) = g(f(x)) \Rightarrow h'(x) = g'(f(x))f'(x)$$

Weitere Ableitungsregeln:

- $f(x) = \sqrt{x}$ $f'(x) = \frac{1}{2\sqrt{x}}$
- $f(x) = \sqrt{g(x)}$ $f'(x) = \frac{g'(x)}{2\sqrt{g(x)}}$
- $f(x) = \frac{1}{x}$ $f'(x) = -\frac{1}{x^2}$
- $f(x) = \frac{1}{g(x)}$ $f'(x) = -\frac{g'(x)}{g^2(x)}$
- $f(x) = \ln x$ $f'(x) = \frac{1}{x}$
- $f(x) = \ln(g(x))$ $f'(x) = \frac{g'(x)}{g(x)}$
- $f(x) = \log_a x$ $f'(x) = \frac{1}{x \ln a}$
- $f(x) = \log_a g(x)$ $f'(x) = \frac{g'(x)}{g(x) \ln a}$
- $f(x) = e^x$ $f'(x) = e^x$
- $f(x) = e^{g(x)}$ $f'(x) = g'(x)e^{g(x)}$
- $f(x) = a^x$ $f'(x) = a^x \ln a$
- $f(x) = a^{g(x)}$ $f'(x) = g'(x)a^{g(x)} \ln a$
- $f(x) = \sin x$ $f'(x) = \cos x$
- $f(x) = \sin(g(x))$ $f'(x) = g'(x) \cos(g(x))$
- $f(x) = \cos x$ $f'(x) = -\sin x$
- $f(x) = \cos(g(x))$ $f'(x) = -g'(x) \sin(g(x))$
- $f(x) = \tan x$ $f'(x) = \frac{1}{\cos^2 x}$